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Holographic evolution of gauge couplings

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Abstract: We study the gauge coupling evolution of a unified theory in the compact Randall-Sundrum model with gauge bosons propagating in the bulk. One-loop corrections in AdS are interpreted in the 4d dual theory as the sum of two contributions: CFT insertions subleading in a $1/N$ expansion and loops of the additional particles coupled to the CFT. We have calculated the scalar loop correction to the low energy gauge couplings both in scenarios where the GUT symmetry is broken by boundary conditions and with the Higgs mechanism. In each case our results are what expected from the holographic dual theory.

Keywords: Extra Large Dimensions, AdS-CFT and dS-CFT Correspondence, GUT.

Contents

1. Introduction

Grand Unified Theories (GUT) with extra dimensions can address some of the longstanding problems of their 4-dimensional (4d) counterparts, while maintaining their virtues. The doublet-triplet splitting problem, for example, is elegantly solved in models where the GUT symmetry is broken by the boundary conditions of the gauge fields in the extra dimensions and not through a Higgs mechanism [1, 2]. The more and more stringent limits on proton decays [3], which are recently getting the minimal 4d SU(5) supersymmetric model into trouble [4], are satisfied in 5-dimensional (5d) extensions [5].

Much attention has been devoted to the case of flat extra dimensions. Here physics appears 4-dimensional in every experiment with typical energies below the inverse radius scale $1/R$, and therefore the running of gauge coupling constants is logarithmic as usual. At higher energies however, nature becomes truly extra dimensional and gauge couplings increase with energy following a power law. By simple dimensional analysis, the logarithmic running comes from the evolution of boundary operators and is associated with a logarithmic divergence, while the power law dependence on external momentum reflects a power divergence in the bulk gauge kinetic term. A power law increase with energy of the coupling constants seems an attractive way to obtain unification of the elementary forces at much lower scales than the usual GUT models [6]. Even more attractive if one speculates on scenarios with quantum gravity at the TeV, in which case unification of strong and weak interactions with gravity seems a realistic ambition. Unfortunately, even if one postulates a sufficient separation of scales between $1/R$ and the cutoff Λ to have an extra-dimensional field theory regime, power law evolution comes together with power threshold corrections, which represent the dominant effect, spoiling completely the predictability. The situation is even worse if one demands unification to occur at the cutoff scale, because this is right the energy domain in which effective field theory breaks down and perturbation theory becomes unreliable. Therefore, if one is so ambitious to insist on predictive schemes, unification has to occur as a result of the slow logarithmic running. This means that the paradigm of a desert between the electroweak scale and a high energy GUT scale $1/R \sim 10^{16}$ GeV, is still valid. A very useful tool in this case is the effective field theory approach of Weinberg [7]: a matching is performed at energy $\mu \sim 1/R$, between the full extra-dimensional GUT theory and a 4d theory with only the Standard Model (SM) gauge degrees of freedom. The heavy GUT states, which have masses of the order of $1/R$, are integrated out and contribute with calculable threshold corrections [8].

The picture drastically changes if we depart from the assumption of flatness. A very interesting situation is the case of just one compact extra dimension, in which the metric is that of Anti-deSitter (AdS) space: the Randall-Sundrum (RSI) model [9]. This model has been originally constructed to address the hierarchy problem, with only gravity propagating in the bulk and the SM confined on the TeV brane. However, its many surprising features have led many groups to explore the possibility of realizing a GUT theory in this warped geometry, with gauge bosons propagating in the bulk [10]–[14]. In RSI, physics appears 5-dimensional at energies higher than the AdS curvature k (which is taken to be of the order of the Planck scale), when everything goes as in the flat limit. Below this scale and down to the weak scale, there is a huge range of energies in which the model is conjectured to be dual to a 4d conformal field theory [15], along the prescription of the AdS/CFT correspondence [16, 17, 18]. This duality allows us to infer that gauge couplings run logarithmically until very high energy, even if new GUT physics, namely the Kaluza-Klein (KK) resonances of the unified gauge bosons, appears at the TeV scale revealing the unified character of the fundamental forces [10]. No surprise then that an effective theory description à la Weinberg does not exist beyond the TeV: changing the GUT group and its breaking modifies the properties of the Conformal Field Theory (CFT) and consequently the evolution of gauge couplings until unification, not only some minor threshold corrections.

In [11] the one-loop correction to the low energy coupling was computed in RSI for a non-abelian gauge theory, employing a momentum cutoff which depends on the fifth dimension. In [13], using dimensional regularization, the case of massless scalar QED was considered, while in [14] also the massive case has been studied, adopting a Pauli-Villars regulator. In this work, we further study the scalar QED case with the computation of the gauge field zero-mode propagator in 5d for different choices of boundary conditions and for a generic scalar bulk mass. In doing that, we choose dimensional regularization that, we believe, is the most economical and transparent regulator which preserves the symmetries of the AdS background. All our results are compatible with what the holographic duality requires. This is sufficient to discuss the scalar contribution to gauge coupling evolution in different GUT scenarios where the unified group is broken by the boundary conditions or through a Higgs mechanism.

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Section 2 is devoted to understand the meaning of the evolution of gauge couplings in AdS space. Planck brane correlators are the only meaningful observables at energies higher than the TeV scale, while mode-by-mode quantities become strongly coupled. We explicitly show how to interpret loop corrections to these observables from the holographic point of view. In section 3 we present the calculation of the scalar loop correction to the low energy gauge couplings, for generic boundary conditions and mass. We leave all the computational details to the appendix. We use these results in section 4 to discuss various mechanisms of GUT symmetry breaking pointing out the agreement with the holographic interpretation. Conclusions are drawn in section 5, where we comment on possible phenomenological scenarios.

2. Holographic interpretation of the running

The Randall-Sundrum model with two branes [9] is simply given by a slice of AdS space with metric

$$ds^2 = \frac{L^2}{z^2}\left(dx^\mu dx^\nu \eta_{\mu\nu} - dz^2\right) \qquad k \equiv \frac{1}{L}\,, \tag{2.1}$$

bordered by two branes respectively at $z = z_0 = 1/k$ (Planck brane) and at $z = z_1 \sim \mathrm{TeV}^{-1}$ (TeV brane). The attractive feature of the model is the possibility to solve the hierarchy problem through a gravitational red-shift of energy scales. Many puzzles of this geometry are better understood through the AdS/CFT correspondence [16, 17, 18]: the holographic dual of this model [15] is a quasi-conformal, strongly coupled 4d theory coupled to 4d gravity. The TeV brane describes a spontaneous breaking of the conformal invariance [19, 20], so that any field living on it is seen, in the dual picture, as a bound state of the CFT itself. Also all the KK modes are seen as CFT condensates, similarly to the resonances of QCD around the GeV scale.

In the following, we consider GUT models where the gauge bosons of the unified group propagate in the AdS bulk and study how this reflects on the low energy gauge couplings. At energies much greater than the TeV scale, the KK states become strongly coupled. Nevertheless, if we restrict to the study of inclusive quantities, given by Green functions on the Planck brane, we can reach energies as high as the Planck scale without entering a strong coupling regime [19, 20] (see also [13]). This is possible because of the exponential die-off of the propagators in the bulk, $G \sim e^{-\sqrt{p^2}z}$ at distances $z \gtrsim p^{-1}$, which makes the high energy processes on the Planck brane insensible of what is going on deep inside AdS: the local cut-off for an observer living on the Planck brane is given by the AdS curvature k. The importance of these inclusive quantities is clear also from the holographic point of view, having the brane-brane correlators a simple 4-dimensional meaning. In our case, the gauge propagator between two points on the Planck brane tells us the strength of the gauge interaction in the 4-dimensional dual theory and it remains perturbative despite the fact that the KK gauge bosons become strongly coupled above the TeV. This dual picture allows us to understand why the gauge coupling running is still logarithmic above the TeV scale: the CFT composites become broader and broader and the true degrees of freedom emerge, but their contribution to the running still remain perturbative and 4-dimensional,

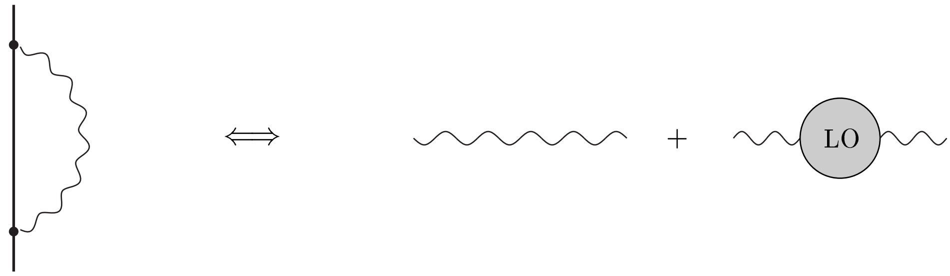

Figure 1: The brane-brane correlator in AdS corresponds holographically to the free gauge propagator corrected by the LO contribution in $1/N$ of the CFT (of order $\sim \mathcal{O}[N^2(\alpha/4\pi)]$ with respect to the tree level). The grey circle represents the $\langle JJ \rangle$ insertion.

i.e. logarithmic. In a unified model, brane-brane gauge correlators for different groups are the same much above the unification scale and this may happen in a regime ($E \gg$ TeV) in which only boundary correlators make sense.

At energies much greater than the TeV scale, but smaller than the AdS curvature k, the tree level Planck brane-brane gauge propagator is given by [10]

$$G(q) = \frac{g_5^2/L}{q^2(\log(2k/q) - \gamma)}\,, \tag{2.2}$$

where γ is the Euler-Mascheroni constant. The holographic interpretation of this formula is quite simple [19]. It just describes the corrections to a 4d vector propagator given by $\langle JJ \rangle_{\rm CFT}$ insertions, where J is the CFT current coupled to the gauge boson (see figure 1). Conformal invariance tells us that $\langle J(p)J(-p)\rangle \propto p^2 \log p^2$, so that the logarithmic running (2.2) follows. It is worthwhile noting that this CFT running is simply described by the tree level AdS propagator and it is common to any gauge group. It follows that, whatever GUT symmetry breaking mechanism we choose, *the leading CFT contribution to the running is always GUT invariant.* From eq. (2.2) we see that the CFT gives a positive contribution to the beta-function $b_{\rm CFT} = 8\pi^2 L/g_5^2 \sim N^2$, where N is the number of colors of the conformal theory [21]. This should be large to ensure that the non-renormalizable 5d gauge theory makes sense: the AdS curvature k must be much smaller than the cut-off scale $\Lambda = 24\pi^3/g_5^2$ or, equivalently, the number N of colors should be large.

2.1 Radiative corrections to brane correlators

Additional contributions to the running of the gauge couplings come from loop corrections to the brane-brane propagator. It is therefore natural to ask what is the holographic interpretation of these loops. For example, what is the 4-dimensional counterpart of the vacuum polarization due to a bulk scalar? In the limit in which we remove the Planck brane, obtaining a complete AdS space, we know that the dual picture is simply a CFT. In this case, bulk loops are interpreted as corrections to the CFT correlators, subleading in a $1/N$ expansion [22]. Concerning the scalar loop correction to the gauge two-point function, we would find a modification of the $\langle JJ \rangle$ CFT correlator. As the dependence of this correlator on the 4d momentum is fixed by conformal invariance, only the coefficient in front receives $1/N$ corrections.

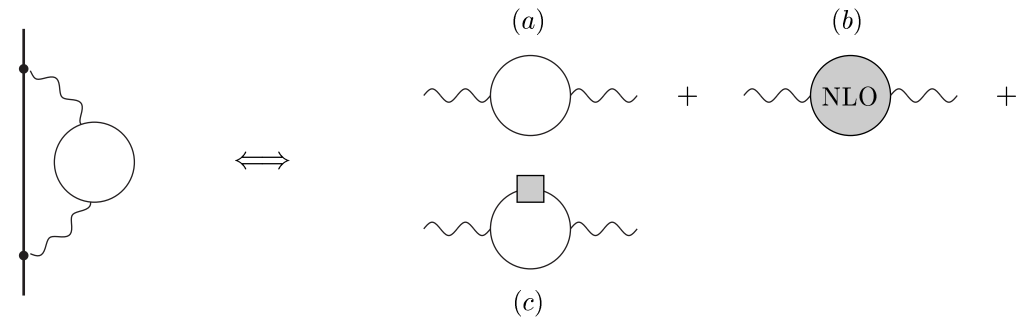

Figure 2: The one-loop (rainbow) scalar correction to the brane-brane correlator in AdS corresponds holographically to three different diagrams: a 4d scalar loop graph (a), the same diagram with the scalar propagator corrected by the CFT (c), and the NLO contribution in $1/N$ of the CFT (b). Diagrams (a), (b) are both $\mathcal{O}(\alpha/4\pi)$ with respect to the tree level; diagram (c) is negligible because the scalar coupling to the CFT is $M_{\rm Pl}$-suppressed. The grey circle (square) represents the $\langle JJ \rangle$ ($\langle OO \rangle$) insertion. A similar holographic interpretation holds for the seagull diagram.

What changes if we add the Planck brane? The rough picture is the following. Cutting off the part of AdS space near its boundary corresponds to a UV modification of the CFT, which is now smeared over a distance of order k^{-1}: degrees of freedom of shorter wavelength have been integrated out. Moreover the 4d role of fields living in AdS space changes. In the full AdS case they are not dynamical from the 4d point of view: their boundary behaviour at infinity just acts as a source for the corresponding operator of the CFT. With the addition of the Planck brane, bulk fields become dynamical also from the 4d viewpoint, as we must integrate over their boundary value on the brane.

We thus expect that radiative corrections to brane correlators in presence of the Planck brane describe not only $1/N$ subleading CFT terms, but the additional contribution of the 4d fields made dynamical by the introduction of the brane. If we have a scalar field in AdS cut by the Planck brane, the 4d theory contains a dynamical scalar, coupled to the CFT through an operator $O(x)$, which has dimension 4 if the scalar is massless. Loops of this 4d scalar will enter the running of the gauge couplings.

As depicted in figure 2, the one-loop AdS correction corresponds to the sum of different terms: the contribution from the 4d scalar (a), whose propagator gets itself a CFT correction (c), and the NLO CFT insertion (b). It is worth noting that the various terms can be arranged in a double expansion: the first is the standard series in powers of $(\alpha/4\pi)$, the second is the expansion of the CFT correlators in powers of $1/N$. The two expansions are related, as the holographic prescription tells us that $1/N^2 \sim g_5^2/16\pi^2 L$. Diagrams 2$(a)$ and 2(b) are of order $\mathcal{O}(\alpha/4\pi)$ with respect to the tree level; diagram 2(c) is completely negligible in this case, being the CFT coupled to the 4d scalar only through $M_{\rm Pl}$-suppressed operators. The corresponding diagram in the case of vector boson loops is $\mathcal{O}[N^2(\alpha/4\pi)^2]$, but still subleading with respect to the other two contributions, as 4d perturbativity requires $N^2(\alpha/4\pi) \ll 1$.

We can look at the contribution (b) and (a) in figure 2 as coming respectively from the limiting case of a 5d loop deep inside AdS or close to the Planck brane. This is quite intuitive, as the 4d scalar field comes from the integration over the boundary conditions on the Planck brane. In the complete AdS case, the boundary values $\phi_0, A^0_\mu, g^0_{\mu\nu}$ for the various fields at infinity act as sources for the corresponding operators in the CFT [18]:

$$\left\langle e^{-\int d^4x\, O(x)\phi^0(x)+J^\mu(x)A^0_\mu(x)+T^{\mu\nu}(x)g^0_{\mu\nu}(x)}\right\rangle_{\rm CFT} = e^{-S_{\rm AdS}(\phi^0,A^0_\mu,g^0_{\mu\nu})}\,. \tag{2.3}$$

The right hand side of this equation must be regularized [18], and this procedure leads us closer to the truncated AdS case we are interested in. The standard procedure is to limit the z integration to $z > \epsilon$ (which corresponds to introducing an explicit UV cut-off on the CFT), add a proper local counterterm action (divergent for $\epsilon \to 0$) function of $\phi^0, A^0_\mu, g^0_{\mu\nu}$ and their derivatives, and then take the limit $\epsilon \to 0$. In the case with only a scalar field, eq. (2.3) becomes

$$\left\langle e^{-\int d^4x\, O(x)\phi^0(x)}\right\rangle_{\rm CFT} = \lim_{\epsilon\to 0} e^{-S_{\rm AdS}(\phi^0,\epsilon)}e^{-S_{\rm count}(\phi^0,\epsilon)}\,. \tag{2.4}$$

Suppose now not to perform the final limit, keeping an explicitly truncated AdS space. As we have integrated out a portion of space which corresponds to the UV of the CFT, we expect this to correspond to a smearing procedure in which fast modes are integrated out [15, 23, 24].[1] At this stage, the scalar loop correction in AdS of figure 2 gives a subleading contribution to the $\langle JJ\rangle_{\phi_0}$ CFT correlator in the external background ϕ_0.

The last step to get the Randall-Sundrum scenario is to integrate over the boundary values $\phi^0, A^0_\mu, g^0_{\mu\nu}$, which become dynamical fields, introducing a generic brane action $S_{\rm bound}(\phi_0)$. Consider for instance a brane action with only a kinetic term proportional to an arbitrary parameter ξ:

$$S_{\rm bound}(\phi_0) = \frac{\xi}{k}\int_{\rm brane} d^4x\sqrt{g}\,\partial_\mu\phi\partial_\nu\phi^\dagger g^{\mu\nu}\,. \tag{2.5}$$

By varying ξ one changes the kinetic term of the 4d scalar and therefore the relative importance between its loop contribution (figure 2(a)) and the CFT correction (figure 2(b)).[2] In the limit $\xi \to +\infty$, the 4d scalar is frozen out and we are left with the CFT correction; the same result holds by choosing Dirichlet boundary conditions on the Planck brane. From these considerations, it should be clear the strict connection between boundary terms in AdS and the 4d scalar mode. Moreover, all the features of the AdS bulk reflect on the CFT. In particular, if the GUT symmetry is unbroken in the bulk, the CFT is GUT-preserving at all orders.

[1]The counterterm action contains an infinite series of increasing dimension, properly suppressed by powers of k: at energies of order of the AdS curvature the theory becomes non-local. In the following we concentrate on the lower dimension operator $(\partial\phi_0)^2$.

[2]Note however that, if the integration over the boundary conditions were done after the addition of the counterterm $S_{\rm count}(\phi_0)$ (eq. (2.4)), the theory would be ill-defined. This is expected because, in this case, the dependence of the 4d action on ϕ_0 is given by the two point function $\langle OO\rangle_{\rm CFT}$ of the corresponding operator [18]: $S(\phi^0) \propto \int d^4x d^4x' \phi^0(x)\phi^0(x')/|x-x'|^8$. This implies that, if the field ϕ_0 is seen as dynamical, it has a non-local kinetic term of the form $q^4\log q$. This property can be used to find $S_{\rm count}(\phi_0)$ at leading order. To fix it we *require* that the 5d brane-brane propagator goes as $1/(q^4\log q)$. It is easy to obtain that this happens for $\xi = -1$. For $\xi > -1$ the integration over the boundary conditions is well defined as the 4d scalar has a conventional kinetic term, with the correct sign.

2.2 The CFT contributions

We now concentrate on the pure CFT corrections, as if we had pushed the Planck brane to infinity, recovering the complete AdS space. We have shown that at leading order the $\langle JJ\rangle$ correlator does not distinguish among the unbroken subgroups of a unified theory, while at NLO the CFT correction is GUT invariant or not depending on the mechanism we choose to break the GUT symmetry. If the symmetry is broken by the boundary conditions, the AdS bulk remains GUT invariant as well as the dual CFT.[3] In this case, at subleading order the CFT still gives a common running to all the unbroken subgroups.

Another possibility is that the unified theory is broken in the bulk, through a vev of a charged scalar Σ. If the expectation value of the scalar is constant along the fifth dimension, the conformal symmetry is still unbroken (all AdS isometries are preserved) but the GUT symmetry is not.[4] In the holographic theory, we have turned on an operator O_Σ coupled to the 4d Σ scalar, transforming under the GUT symmetry, which therefore results spontaneously broken.[5] In this scenario, GUT-breaking corrections to Planck brane propagators correspond to analytic or non-analytic operators, involving Σ, in the 5d effective action. The contribution due to analytic operators is not calculable, and can be only estimated through a naive dimensional analysis. For instance, in the case of the ΣFF operator, naive dimensional analysis gives a ratio between the $\mathcal{O}(1)$ and $\mathcal{O}(N^2)$ corrections to the CFT beta-function:

$$\frac{b_{\rm CFT}^{\rm NLO}}{b_{\rm CFT}^{\rm LO}} \sim \frac{g_5\langle\Sigma\rangle}{\Lambda} = \frac{M_{\rm GUT}}{\Lambda}\,, \tag{2.6}$$

where $\Lambda = 24\pi^3/g_5^2$ is the 5d cut-off. The second equality relates this ratio to the mass of the 4d GUT gauge bosons: $M_{\rm GUT} = g_5\langle\Sigma\rangle$. If we want unification to occur in the regime in which the holographic dual makes sense, we have to require $M_{\rm GUT} \ll k$, so that the ratio (2.6) must be quite small.

From the 4d point of view these corrections to the $\langle JJ\rangle_{\rm CFT}$ correlator are due to the multiple Green functions involving the additional operator O_Σ:

$$\langle O_\Sigma O_\Sigma \cdots JJ\rangle_{\rm CFT}\,. \tag{2.7}$$

For example the ΣFF AdS vertex gives at tree level a CFT correlator $\langle O_\Sigma JJ\rangle_{\rm CFT}$ [21]; turning on the O_Σ operator thus modifies the current-current correlators in a non GUT-invariant way. All these corrections are suppressed by powers of $1/N$ and $\lambda \equiv \langle\Sigma\rangle/k^{3/2}$, where λ is the coupling constant of the operator O_Σ in the 4d picture.

Notice that the non-calculability due to higher-dimension operators in AdS is here reflected into an incalculable CFT beta-function. Nevertheless predictability is retained

[3] If the boundary conditions on the TeV brane break the GUT symmetry, in the dual picture we have a spontaneous breaking of the symmetry at the TeV scale, together with the conformal breakdown. Still, the running above the TeV scale remains GUT invariant, as we will explicitly check in the following sections.

[4] We may also consider operators which break the conformal symmetry, corresponding to massive scalars. In this case the scalar profile will not be constant in AdS.

[5] In presence of the Planck brane, the GUT symmetry breaking is spontaneous, due to the vacuum expectation value of the Σ field. In the complete AdS case one just turns on the corresponding operator O_Σ, causing an explicit breaking.

if we assume that the AdS picture is weakly coupled so that the perturbative expansion makes sense, with higher dimension operators properly suppressed by powers of the 5d cut-off Λ. In this case, CFT contributions which distinguish among the unbroken subgroups are suppressed with respect to the GUT invariant leading CFT running. In turn, this leading contribution cannot be too large if we want to build a phenomenologically viable model.

The reason for this is quite simple and it is a general problem of unification in this kind of models: the GUT-invariant CFT running would imply, for $N \gg 1$, that *at low energy we should see nearly* SU(5)*-invariant couplings:* $\Delta\alpha/\alpha \ll 1$. Alternatively: if N is too large, we meet the strong coupling regime before reaching the unification scale. It is easy to deduce a limit on N from the requirement of perturbativity at the GUT scale (which we take to be of the order of the standard one $M_{\rm GUT} \sim 10^{16}\,{\rm GeV}$): $N^2(\alpha_{\rm GUT}/4\pi) \ll 1$, where the N^2 factor comes from the number of CFT states. From this we obtain:

$$b_{\rm CFT} \ll \frac{2\pi\alpha_i^{-1}(\,{\rm TeV}) - b_i^0 \log M_{\rm GUT}/\,{\rm TeV}}{1 + \log M_{\rm GUT}/\,{\rm TeV}} \sim 8\,, \tag{2.8}$$

where the numerical bound is obtained for b_i^0 given by the SM matter content. An opposite bound on $b_{\rm CFT}$, or equivalently on N, comes from the requirement of perturbativity in 5d, namely $\Lambda/k\pi \gg 1$, where Λ is the 5d cutoff: the inequality

$$b_{\rm CFT} = \frac{8\pi^2 L}{g_5^2} \gg \frac{1}{3} \tag{2.9}$$

follows.

The general conclusion is that the leading CFT running cannot be much greater than other contributions which separate the unbroken subgroups, coming from additional particles coupled to the CFT. The limit on N is not so strong to spoil the perturbativity of the AdS picture, as we see comparing eqs. (2.8) and (2.9), even if the allowed window is not too wide. This limit on the CFT leading contribution implies, in turn, that subleading corrections, coming from bulk loops and higher dimension operators are negligible with respect to the non-CFT running.

In principle we could discuss the gauge coupling running even in absence of a unified group in the bulk and check if the gauge couplings cross at a certain energy. In this case the CFT contribution to the running of each group is different at leading order, so that the running may be much faster with a consequent lowering of the unification scale [11]. However, the CFT beta-function for the three groups, given at leading order by the three independent gauge kinetic terms, is incalculable, so that no firm prediction seems possible.

Before moving to the explicit calculations, we want to stress an important conceptual difference between the standard models of unification and the ones built in AdS space. In the standard case, Weinberg's approach of effective gauge theory is very useful and it tells us that the details of GUT-symmetry breaking, resulting only in threshold corrections, are not crucial to test unification. Here the situation is different. Modifying the unified gauge group we are at the same time changing the CFT excitations, hopefully around the corner, at the TeV scale. The pattern of symmetry breaking does not influence only the physics at far-away energies, but also the subleading CFT corrections to the running down to the

TeV scale. All this follows from the fact that AdS space describes at the same time the CFT properties and the behaviour of the additional particles coupled to it.

3. The low energy gauge coupling

In this section we present our result for the one-loop scalar correction to the low energy coupling of a U(1) gauge group in the bulk. We leave to the appendix all the computational details, focusing our attention on the holographic interpretation. Once given the main formulae for different boundary conditions of the scalar field, we will able in the next section to discuss various scenarios of GUT symmetry breaking.

In order to regulate the loop divergence we choose the dimensional regularization, which proved to be a powerful scheme also in theories with flat extra-dimensions [25]. In the specific case of the one-loop correction to the zero-mode gauge correlator, it is enough to extend the brane dimension to a generic (complex) value d keeping just one extra dimension. Analogously to the Minkowski case, the isometries of AdS space are clearly preserved. The zero-mode gauge self-energy reads, for external 4d momentum p:

$$\frac{1}{g^2(p^2)} = \frac{\log(z_1/z_0)}{k\, g_5^2} + \Delta_0(\mu) + \Delta_1(\mu) - \Pi(p^2,\mu)\,, \tag{3.1}$$

where μ is the subtraction point and $\Delta_{0,1}(\mu)$ are the coefficients of the gauge kinetic terms localized on the branes. $\Pi(p^2,\mu)$ is the one-loop scalar correction

$$\Pi(p^2,\mu) = -\mu^{4-d} \sum_{\{x_n\}} \int_0^1 dx (2x-1)^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{[q^2 + x_n^2 + c^2(x)]^2}\,. \tag{3.2}$$

Here $c^2(x) = x(1-x)(-p^2)$ and x_n is the mass of the n-th Kaluza-Klein mode of the scalar field (see appendix). Using the technique described in the appendix, it is easy to perform the integration first and then the sum, getting

$$\begin{aligned}\Pi(p^2,\mu) \;=\; \frac{(b_0/2)}{8\pi^2}\Bigg[& -\frac{\alpha}{\epsilon} + \log\left(\sqrt{-p^2}\sqrt{z_0 z_1}\right) + \alpha \log\frac{\sqrt{-p^2}}{\mu} + \\ & + 3\int_0^1 dy\, y\sqrt{1-y^2}\,\log f\left(\frac{iy\sqrt{-p^2}}{2}\right) + \\ & + \alpha\frac{\gamma}{2} + \log\pi - \frac{4}{3}(1+\alpha)\Bigg]\,. \end{aligned} \tag{3.3}$$

With $b_0 = 1/3$ we mean the beta-function of a charged 4d scalar, and $d = 4-\epsilon$. The previous formula is a completely general result, valid for a scalar with arbitrary boundary conditions and mass; in the case of $(\pm\pm)$, $(\pm\mp)$ boundary conditions, one should read $\alpha = \pm 1$, $\alpha = 0$ respectively and choose a function $f = f_{\pm\pm}$, $f = f_{\pm\mp}$, whose expression is given in appendix. In the particular case of a $(++)$ massless scalar, eq. (3.3) coincides with the result of [13].

The zero-mode gauge propagator is an exclusive observable and does not make sense above the TeV where the 0 mode becomes strongly coupled. This means that eq. (3.3) can

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be really trusted only for external momenta $|p| \lesssim$ TeV [13]; at these energies it matches the Planck brane-brane correlator, therefore admitting a simple holographic interpretation. Once the function f in eq. (3.3) is expanded for $z_1|p| \ll 1$, the logarithmic dependence on the momentum p must be the correct one for an infrared log. The logarithmic divergence, represented by the $1/\epsilon$ pole, is the same as in the flat limit (for the latter, see [8]). This was expected, because in the very high energy regime the curvature can be neglected and AdS appears locally flat [13, 14].

In the following we collect the low energy limit $z_1|p| \ll 1$ expression of $\Pi(p^2, \mu^2)$ for all possible choices of boundary conditions in the massless case and for a (++) scalar with AdS bulk mass m. Using the asymptotic expansions of eqs. (6.12), we obtain (subtracting the $1/\epsilon$ divergence and omitting irrelevant constants):

Massless scalar $\quad (1/z_1 \gg |p| > z_0/z_1^2)$**.**

$$\Pi_{++}(p^2, \mu) \simeq \frac{b_0}{8\pi^2}\left[\log\frac{z_1}{z_0} + \log z_0\sqrt{-p^2} - \frac{1}{4}\log \mu z_0 - \frac{1}{4}\log \mu z_1\right] \tag{3.4}$$

$$\Pi_{--}(p^2, \mu) \simeq \frac{b_0}{8\pi^2}\left[\log\frac{z_1}{z_0} + \frac{1}{4}\log \mu z_0 + \frac{1}{4}\log \mu z_1\right] \tag{3.5}$$

$$\Pi_{-+}(p^2, \mu) \simeq \frac{b_0}{8\pi^2}\frac{3}{4}\log\frac{z_1}{z_0} \tag{3.6}$$

$$\Pi_{+-}(p^2, \mu) \simeq \frac{b_0}{8\pi^2}\left[\frac{5}{4}\log\frac{z_1}{z_0} + \log z_0\sqrt{-p^2}\right]. \tag{3.7}$$

Massive (++) scalar $\quad (k \gg m \gg |p|,\ |p| \ll 1/z_1)$**.**

$$\Pi(p^2, \mu) \simeq \frac{b_0}{8\pi^2}\left[\log\frac{z_1}{z_0} + \log m z_0 - \frac{1}{4}\log \mu z_0 - \frac{1}{4}\log \mu z_1 + \frac{m^2 z_0^2}{8}\left(\log\frac{z_1}{z_0} - \frac{1}{2}\right)\right]. \tag{3.8}$$

Massive (++) scalar $\quad (m \gg k \gg 1/z_1 \gg |p|)$**.**

$$\Pi(p^2, \mu) \simeq \frac{b_0}{8\pi^2}\left[\frac{3}{4}\log\frac{z_1}{z_0} + \frac{1}{2}\log\frac{m}{\mu} + \frac{1}{2}z_0\sqrt{m^2}\log\frac{z_1}{z_0}\right]. \tag{3.9}$$

For the (++) scalar these equations agree with the results of [14] if $\mu = k$.

The $\log p$ terms in the previous formulae are the expected infrared logarithms. Holographically they correspond, in the (++) massless case, to the 4d massless mode which runs logarithmically from high scale down to low energy. It can be interpreted as the Goldstone boson of the symmetry $\phi \to \phi + \text{const.}$, which shifts the 5d scalar field by a constant. This symmetry is broken in the (+−) case by the boundary condition on the TeV brane and the Goldstone boson acquires a tiny mass $M \sim z_0/z_1^2 \sim 10^{-4}$ eV. That M should be so small can be understood by the following argument: the 4d scalar couples to the CFT with a $M_{\rm Pl}$ suppressed operator and the analog of the pion decay constant is $f_\pi \sim k$. A Dirichlet boundary condition on the TeV brane implies the breaking of the $\phi \to \phi + \text{const.}$ symmetry with a typical breaking scale $\sim$ TeV. Then a mass follows for the pseudo-Goldstone boson $M^2 \sim \text{TeV}^4/f_\pi^2 \sim \text{TeV}^4/k^2$ (the exact value of the mass can

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be derived as the lightest eigenvalue of the spectrum equation (6.2) given in appendix [6]). We conclude that, for scalar boundary conditions $(+-)$, the holographic theory contains an almost massless 4d scalar contributing to the running of the gauge coupling down to very low energy.[7] This explains the $\log p$ term in Π_{+-}.

Non-analytic operators in the bulk, like $\sqrt{\mathcal{R}}FF$, in a background with a constant scalar curvature $\mathcal{R} \propto k^2$ give a bulk kFF operator which corresponds in $\Pi(p^2, \mu)$ to calculable $\log z_1/z_0$ terms.[8] This local bulk effect is interpreted holographically as a calculable correction to the CFT beta-function. There is also an additional incalculable correction coming from the linear divergence of the bulk gauge kinetic term, but of course this does not appear in dimensional regularization. Assuming an holographic point of view, one can extract this NLO CFT contribution from any of the equations (3.4)–(3.7). Consider for example eq. (3.4): the $\log pz_0$ term is the running of the 4d scalar from the Planck scale down to p; the latter two terms, coming from the log divergence on the AdS boundary, are threshold corrections in the 4d theory at the scales $1/z_0$, $1/z_1$. The remaining contribution, namely the first term in eq. (3.4), is the calculable part of the NLO CFT correction. On the other hand, any of the equations (3.4)–(3.7) does not constitutes by itself an unambiguous test of the holographic interpretation. Such an ambiguity can be resolved only by comparing the different results for the various parities as we will do in the next section when we consider the GUT breaking scenarios. In the massive case, eqs. (3.8),(3.9), there are also contributions of the form $z_0^2 m^2 \log z_1/z_0$, $z_0\sqrt{m^2} \log z_1/z_0$; both are calculable, as they correspond to AdS bulk operators which depend non-analytically on the scalar curvature $\mathcal{R}$ (the former) or on the lagrangian parameter m^2 (the latter). The $z_0\sqrt{m^2} \log z_1/z_0$ terms appear uniquely for a very large value of the mass, $m \gg k$ (see eq. (3.9)). In this limit they represent the contribution of CFT operators of very high dimension $\propto m/k$, while $\log m$ terms can be interpreted only from a 5d point of view: their coefficient $b_0/2$ comes from the running of boundary operators.

It is well known [27] that, in the 5d flat case, gauge kinetic terms on the boundary evolve logarithmically with energy, and their beta-function gets a one-loop contribution from particles living in the bulk. This evolution is intimately connected with a logarithmic divergence. The whole tower of massive KK states contributes to the running on a given boundary with $1/4$ of the beta-function b_0 of the zero-mode, the sign of the effect depending on the parity of the loop fields [8]: including also the zero-mode contribution, one finds $\pm 1/4\, b_0$ if the loop field is $\pm$ on that specific boundary. The logarithmic divergences, together with the associated $\log p/\mu$ terms, cancel in the one-loop correction from a $(\pm, \mp)$ scalar to the zero-mode gauge propagator, summing the contributions at the two boundaries. All these considerations must remain valid in the warped case as well, being

[6]Also ref. [26] has recently pointed out the appearance of such a small eigenvalue in the similar case of a boundary mass term on the TeV brane for the scalar field.

[7]In the case of a vector field, Dirichlet boundary conditions on the TeV brane implies a mass $\sim$ TeV. This is expected, because its coupling with the CFT is dimensionless so that the mass is only logarithmically suppressed by $1/\log(k/\,\mathrm{TeV})$. For a fermion field we obtain $m^2 \sim \mathrm{TeV}^2 \cdot \mathrm{TeV}/k$.

[8]The non-analyticity is a consequence of the fact that we need a term linear in k, while the metric is a function of k^2. We thank Riccardo Rattazzi for clarifying us this point.

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the divergences the same as in the flat limit. This can be verified looking at eqs. (3.4)–(3.9). In the warped case, the contribution from massive KK states to the running of operators on the boundaries $z = z_0, z_1$ will freeze out at the typical local scale $1/z_0$, $1/z_1$. This gives the $\log \mu z_0$, $\log \mu z_1$ terms in $\Pi(p^2, \mu)$ which are the counterpart of the $\log \mu R$ terms of the flat case. A further source of $\log z_{0,1}$ terms might be finite non-local operators which will be in general present in the 5d effective action. Indeed, the dependence on $z_{0,1}$ of the function f in eq. (3.3), is quite complicated before taking the limit $z_0 \ll z_1$. Only when the is a large separation of scales $z_0 \ll z_1$, we recover the simple expression of eqs. (3.4)–(3.7) required by the holographic interpretation.

Concerning the holographic interpretation of the brane kinetic terms in AdS, they correspond to adding a constant term to the 4d inverse coupling $1/g^2(p^2)$, shifting its Landau pole [19]. In other words, it is a modification of the 4d theory at a scale corresponding to the position of the brane in AdS. There is therefore no connection between boundary terms in AdS and log evolution in the holographic theory. It is remarkable that in the flat case all the logarithmic running comes from boundary operators, while the main logarithmic running in the 4d theory dual to RSI comes from the AdS bulk.

4. GUT breaking: the holographic point of view

Armed with the previous results, we discuss now different mechanisms of breaking the GUT symmetry in AdS, either through suitable boundary conditions for the gauge fields, or turning on the vev of a scalar field in the bulk. We consider for simplicity the particular case of an SU(5) group in the bulk broken down to SU(3) × SU(2) × U(1) and we study the loop correction to the low energy couplings given by a scalar multiplet in the fundamental representation. It is understood that the results have a general validity.

4.1 GUT breaking through boundary conditions

Let us consider first the case in which the GUT symmetry is reduced at low energy by the boundary conditions. We assume that the SU(3) × SU(2) × U(1) gauge bosons A^a_μ have always parity $(++)$, while the X, Y bosons $A^{\hat{a}}_\mu$ can be $(\pm,\mp)$ or $(--)$: SU(5) is broken on the TeV or Planck brane, or both. The relative parities of the doublet and triplet components of the scalar 5-plet φ in the bulk are fixed by gauge invariance. We choose $\varphi_2 = (++)$ for the doublet component and this forces $\varphi_3 = (\pm,\mp)$, $(--)$ for the triplet when $A^{\hat{a}}_\mu$ are $(\pm,\mp)$, $(--)$ respectively.

GUT breaking on the TeV brane.

$$\varphi = \begin{bmatrix} \varphi_2(++) \\ \varphi_3(+-) \end{bmatrix} \qquad \text{for} \quad A^{\hat{a}}_\mu(+-) \quad A^{\hat{a}}_5(-+)\,.$$

A theory with a gauge group SU(5) in pure AdS is dual to a 4d CFT with a *global* SU(5) invariance. Putting the Planck brane and imposing + conditions for the gauge bosons corresponds, in the holographic theory, to gauge the global symmetry. Let us now insert the TeV brane demanding − parity for the X, Y (and + for the A^a_μ) gauge fields.

This deformation in AdS implies in the 4d picture a spontaneous breaking of SU(5) down to the SU(3) $\times$ SU(2) $\times$ U(1) subgroup at the TeV: the X, Y bosons acquire TeV masses through the Higgs mechanism and the CFT resonances are not SU(5) invariant. At energies higher than the TeV, however, the Planck brane-brane correlator does not probe the GUT breaking on the TeV brane and the holographic theory must appear fully SU(5) invariant. As a consequence, we expect a GUT-invariant running of the SU(3) $\times$ SU(2) $\times$ U(1) gauge couplings g_i, $i = 1, 2, 3$, from the TeV up to higher energies. This is indeed what we found computing the contribution of the massless 5-plet scalar (for $1/z_1 \gg |p| \gg z_0/z_1^2$):

$$\frac{1}{g_i^2(p^2)} = \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2} - b_5\right] + \Delta_0\left(\frac{1}{z_0}\right) + \Delta_1^i\left(\frac{1}{z_1}\right) - \frac{b_5}{8\pi^2}\log z_0\sqrt{-p^2} - \\ - \frac{1}{8\pi^2}\left[\frac{b_2^i}{2}\left(-\frac{1}{\epsilon} + \frac{\gamma}{2} - \frac{8}{3}\right) - \frac{b_3^i}{2}\left(2\log 2 + \frac{8}{3}\right)\right]. \tag{4.1}$$

We denote with $b_{2,3}^i$ the beta-functions of a 4d scalar doublet, triplet respectively and with b_5 the SU(5)-invariant beta-function. We recognize in the previous formula (fourth term) the contribution of the holographic 5-plet (a massless doublet and a triplet with a tiny mass $\sim$ TeV$^2/k$), and the CFT contribution at NLO in $1/N$ (first term). Both are SU(5) invariant as expected.[9] Notice that for this to happen, we had to evaluate the boundary couplings $\Delta_{0,1}(\mu)$ on the Planck and TeV branes at $\mu = k$, TeV respectively. This is quite natural as these boundary terms $\Delta_{0,1}$ correspond holographically to threshold corrections at the scales $1/z_0$, $1/z_1$. Had we evaluated, for instance, the TeV boundary term at $\mu = k$, a fake SU(5)-breaking effect would have been introduced, coming from the SU(5) non-invariant evolution of $\Delta_1^i(\mu)$. The logarithmic divergence is canceled with an SU(5) non-invariant counterterm on the TeV brane: the only source of differentiation among the three couplings g_i comes from $\Delta_1^i(1/z_1)$ and from some finite scheme-dependent terms absorbable in Δ_1^i. Changing the value of the latter, corresponds holographically to modify the Higgs mechanism responsible for the SU(5) breaking. How much the g_i depart from a common value below the TeV depends therefore on the unknown value of $\Delta_1^i(1/z_1)$. It is clearly an important phenomenological question to estimate this contribution in some way. One can advocate a plausible strong coupling hypothesis [8] assuming that the $\Delta_1^i(\mu)$ are sufficiently small when the gauge dynamics becomes strongly coupled. On the TeV brane this happens at energies $\mu \gtrsim$ TeV, confirming that the choice of the scale $\mu = 1/z_1$ for Δ_1 is the correct one.

GUT breaking on both the TeV and Planck brane.

$$\varphi = \begin{bmatrix} \varphi_2(++) \\ \varphi_3(--) \end{bmatrix} \qquad \text{for} \quad A_\mu^{\hat{a}}(--)A_5^{\hat{a}}(++)\,.$$

If SU(5) is broken by the Planck brane boundary conditions, the holographic theory does not have X, Y bosons. Even if the CFT has a global SU(5) invariance (see section 2),

[9] At very low energies, $|p| < z_0/z_1^2$, the triplet contribution stops and the running becomes different for the three SU(3) $\times$ SU(2) $\times$ U(1) couplings g_i.

only the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry is gauged. In the holographic theory we thus find, in addition to the CFT sector, the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge fields and an elementary doublet scalar. Inserting the TeV brane in AdS and demanding a $-$ parity for the $A_\mu^{\hat{a}}$, the global SU(5) invariance of the CFT is spontaneously broken at the TeV to the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ subgroup. The corresponding Goldstone bosons can be identified with the zero modes of $A_5^{\hat{a}}$, which appear in the dual theory as scalar excitations of the CFT with the same quantum numbers of the XY bosons. From the holographic point of view, we thus expect an SU(5)-breaking running up to the Planck scale given by the scalar doublet, while the CFT does not contribute to the differential running. Indeed the explicit calculation gives:

$$\frac{1}{g_i^2(p^2)} = \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2}-b_5\right]+\Delta_0^i\left(\frac{1}{z_0}\right)+\Delta_1^i\left(\frac{1}{z_1}\right)-\frac{b_2^i}{8\pi^2}\log z_0\sqrt{-p^2}\,- \\ -\frac{1}{8\pi^2}\left[\frac{(b_2^i-b_3^i)}{2}\left(-\frac{1}{\epsilon}+\frac{\gamma}{2}\right)-\frac{b_3^i}{2}\log 2-b_2^i\frac{4}{3}\right]. \tag{4.2}$$

This equation gives a non-ambiguous test of the holographic interpretation: the 4d scalar doublet gives a differential running up to the scale k. This effect cannot be falsified by the SU(5)-invariant running of the CFT.

An important observation is in order at this point. From the holographic point of view, there is no reason at all why the different g_i couplings should unify at the Planck scale. Indeed, in the holographic theory SU(5) is just a global symmetry of the pure CFT sector, only the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ group is gauged. This is in sharp contrast with the case of SU(5) broken only by the TeV brane: in that case, there is a Higgs mechanism in 4d reducing the GUT group at the TeV. No analogous mechanism arises here at the Planck scale. Moreover, from the 5d point of view, the situation at energies around k is similar to the flat case: there is really no exact unification of the gauge couplings just because there is no unified symmetry on the boundaries. As in the flat limit, however, one can estimate the threshold corrections, represented in AdS by the boundary term $\Delta_0^i(\mu)$, to be small if evaluated at a scale $\mu\sim 1/z_0$ close to the strong dynamics regime. In this sense, we recover an approximate unification of the couplings g_i at the Planck scale.

GUT breaking on the Planck brane.

$$\varphi=\begin{bmatrix}\varphi_2(++)\\ \varphi_3(-+)\end{bmatrix} \qquad \text{for} \quad A_\mu^{\hat{a}}(-+)A_5^{\hat{a}}(+-)\,.$$

The GUT symmetry is still broken on the Planck brane but no more on the TeV, so that the holographic picture is much similar to the previous case. Inserting a TeV brane and demanding a $+$ parity for the $A_\mu^{\hat{a}}$, it means that SU(5) remains a global symmetry of the CFT: the CFT resonances can be arranged in exact SU(5) multiplets. As in the previous case we expect that the only source of SU(5) breaking comes from the scalar doublet. Indeed we obtain

$$\frac{1}{g_i^2(p^2)} = \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2} - b_5\right] + \Delta_0^i\left(\frac{1}{z_0}\right) + \Delta_1\left(\frac{1}{z_1}\right) - \frac{b_2^i}{8\pi^2}\log z_0\sqrt{-p^2} - \\ - \frac{1}{8\pi^2}\left[\frac{b_2^i}{2}\left(-\frac{1}{\epsilon} + \frac{\gamma}{2} - \frac{8}{3}\right) + \frac{b_3^i}{2}\log 2\right]. \tag{4.3}$$

4.2 GUT breaking with a bulk vev

A different mechanism to break the GUT symmetry is the standard Higgs mechanism. Let us suppose that a massless scalar field Σ, propagating in the bulk, acquires a vacuum expectation value $\langle\Sigma\rangle$ constant along the fifth dimension. In the following we assume that Σ and all the other bulk fields have $(++)$ boundary conditions. This vev splits the masses of the GUT multiplets, giving, for example, a (bulk) mass $m \sim g_5\langle\Sigma\rangle$ to the triplet of our scalar φ, leaving the doublet massless. An interesting possibility is that $k \gg m \gg$ TeV so that the one-loop correction to the low energy couplings reads:

$$\frac{1}{g_i^2(p^2)} = \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{k\,g_5^2} - b_5 - b_3^i\frac{m^2z_0^2}{8}\right] + \\ + \Delta_0\left(\frac{1}{z_0}\right) + \Delta_1\left(\frac{1}{z_1}\right) - \frac{b_2^i}{8\pi^2}\log\frac{\sqrt{-p^2}}{m} - \frac{b_5}{8\pi^2}\log mz_0 - \\ - \frac{1}{8\pi^2}\left[\frac{b_5}{2}\left(-\frac{1}{\epsilon} - +\frac{\gamma}{2}\right)\frac{4}{3}b_2^i - \frac{b_3^i}{2}\log 2 - -b_3^i\frac{m^2z_0^2}{16}\right]. \tag{4.4}$$

In the 4d dual picture the gauge symmetry is spontaneously broken. The 4d doublet and triplet scalars take different masses (the triplet has a mass $\sim m/\sqrt{2}$, corresponding to the lowest eigenvalue of the 5d KK tower): their contribution can be recognized in eq. (4.4). The vev of the Σ field implies, as discussed in section 2, that the CFT is not SU(5) invariant. Therefore, we expect GUT symmetry breaking terms in the CFT beta-function proportional to m^2/k^2; in fact they appear in the first term of eq. (4.4). While the $\log m$ can be traced back to a calculable and non-analytic operator $(\log\Sigma)FF$ in the AdS effective action, the m^2/k^2 terms come from Σ^2FF operators on the AdS side. As a last remark, we notice that there are no terms in eq. (4.4) linear in m. They would be the counterpart either of an analytic 5d operator ΣFF or of the non-analytic operator $\sqrt{\Sigma^2}FF$. The first one is absent if we impose a $\Sigma \to -\Sigma$ symmetry and the second one shows up only in the flat limit $m \sim g_5\langle\Sigma\rangle \gg k$, as already said in section 3.

5. Conclusions

We have studied the dynamics of gauge interactions in the Randall-Sundrum model with gauge bosons in the bulk, which is conjectured to be dual to a 4d CFT weakly coupled to the corresponding 4d gauge sector. This duality allows us to keep a perturbative control on the model up to Planck scale, if we limit our study to inclusive correlators on the Planck brane. The evolution of the gauge couplings up to high energies in the holographic theory gives an insight of the dynamics in the 5d theory. Bulk loop corrections to brane-brane correlators give both the $1/N$ expansion of the CFT and the ordinary perturbative expansion in powers of the gauge coupling constant.

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Using dimensional regularization, we have calculated the 1-loop correction to the low-energy gauge couplings in 5d due to a bulk scalar with various boundary conditions on the two branes and arbitrary mass. These zero-mode propagators give the 4d holographic couplings at low energy with their evolution from the Planck scale.

The calculations allowed us to study different GUT scenarios where the gauge symmetry is broken either by a Higgs mechanism, or by the boundary conditions. We have checked that in any case the results are compatible with what expected from the holographic dual.

Some general conclusions can be drawn for model building. We have seen that, as the CFT has a positive beta-function, strong limits are obtained if one imposes that the gauge coupling remains perturbative up to a standard GUT scale ($\sim 10^{16}\,\mathrm{GeV}$): roughly speaking, the CFT has not to be dominant with respect to the other contributions, so that large values of N are forbidden. This in turn implies that subleading CFT contribution is typically negligible.

Different phenomenological models are possible. If the Standard Model particles are confined on the Planck brane, supersymmetry is required to stabilize the hierarchy; one reobtains a standard supersymmetric unification, if a spontaneous breaking occurs on the Planck brane [10]. From the 4d point of view, we have just added to the MSSM a GUT-invariant CFT, which just gives a common positive contribution to all the three beta-functions.

As discussed in [11], we can also imagine to put the Standard Model on the TeV brane, in order to solve the hierarchy problem. In this case, proton decay mediated by X,Y Kaluza-Klein bosons with TeV masses must be forbidden; for example by choosing Dirichlet boundary conditions for the broken gauge bosons and requiring additional symmetries for the TeV brane interactions. This breaking of the GUT symmetry through TeV brane boundary conditions is negligible for energies above the TeV scale; additional sources of GUT breaking are therefore required, such as a Higgs mechanism in the bulk or on the Planck brane.

If the only source of symmetry breaking is the choice of boundary conditions on the TeV brane, the unification scale should be at the TeV scale. This could fit well in the framework of $\mathrm{SU}(3)_W$ unification [28], recently readdressed in extra-dimensional inspired models [29], in which the SU(2) and U(1) groups of the Standard Model are embedded into a weak SU(3) around the TeV scale. However, it is likely that this kind of model requires a scale of conformal symmetry breaking too low to be compatible with the strong limits coming from electroweak precision observables [30].

A further possibility is the breaking through Planck brane boundary conditions. In this case, there is no unification in the usual sense, as only the SM gauge bosons exist in the holographic dual . Nevertheless, as in the flat case, an approximate unification at high energies can be justified from a 5d point of view, relying on a strong coupling assumption for the boundary couplings on the Planck brane.

Using both the AdS picture and the 4d dual counterpart, unification of gauge couplings in these warped spaces can be discussed. Only further work will tell us if a viable and compelling model is achievable.

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6. Sums in AdS

We present here the method used to sum the series of eq. (3.2) on the AdS Kaluza-Klein masses. Performing first the integral in eq. (3.2), one find the series

$$S(d)=\sum_{\{x_n\}}\left(x_n^2+c^2(x)\right)^{d/2-2}\,, \tag{6.1}$$

where the summation runs over the entire KK spectrum of the scalar field. Depending on its boundary conditions, the KK masses x_n of a massive scalar field in AdS satisfy the following eigenvalue equations:

$$\begin{aligned} (++):&\quad \frac{j_\nu(x_nz_0)}{y_\nu(x_nz_0)}=\frac{j_\nu(x_nz_1)}{y_\nu(x_nz_1)}; &\quad (--):&\quad \frac{J_\nu(x_nz_0)}{Y_\nu(x_nz_0)}=\frac{J_\nu(x_nz_1)}{Y_\nu(x_nz_1)}\\ (+-):&\quad \frac{j_\nu(x_nz_0)}{y_\nu(x_nz_0)}=\frac{J_\nu(x_nz_1)}{Y_\nu(x_nz_1)}; &\quad (-+):&\quad \frac{J_\nu(x_nz_0)}{Y_\nu(x_nz_0)}=\frac{j_\nu(x_nz_1)}{y_\nu(x_nz_1)}\,. \end{aligned} \tag{6.2}$$

Here J_ν, Y_ν are Bessel functions, $\nu=\sqrt{4+m^2z_0^2}$, with m the 5d mass, and

$$y_\nu(z)=Y_{\nu-1}(z)+\frac{(2-\nu)}{z}Y_\nu(z)\,;\qquad j_\nu(z)=J_{\nu-1}(z)+\frac{(2-\nu)}{z}J_\nu(z)\,. \tag{6.3}$$

Choosing the functions:

$$\begin{aligned} f_{++}(z)&=y_\nu(zz_1)j_\nu(zz_0)-y_\nu(zz_0)j_\nu(zz_1)\\ f_{--}(z)&=Y_\nu(zz_1)J_\nu(zz_0)-Y_\nu(zz_0)J_\nu(zz_1)\\ f_{+-}(z)&=i\left[y_\nu(zz_0)J_\nu(zz_1)-Y_\nu(zz_1)j_\nu(zz_0)\right]\\ f_{-+}(z)&=i\left[y_\nu(zz_1)J_\nu(zz_0)-Y_\nu(zz_0)j_\nu(zz_1)\right]\,, \end{aligned} \tag{6.4}$$

whose zeros are the x_ns, one can rewrite the sum in eq. (6.1) as a complex integral over the contour Γ with $R\to\infty$ (see figure 3):

$$S(d)=\frac{1}{2\pi i}\int_\Gamma dz\,\left(z^2+c^2\right)^{d/2-2}\frac{f'(z)}{f(z)} \tag{6.5}$$

with f one of the functions in eq. (6.4).[10]

[10]Even if the domain of definition of the Bessel functions $J_\nu(z)$, $Y_\nu(z)$ is the z-plane cut along the negative real axis, the functions $f_{\pm,\pm}(z)$, $f_{\pm\mp}(z)$ are single-valued on the entire complex plane.

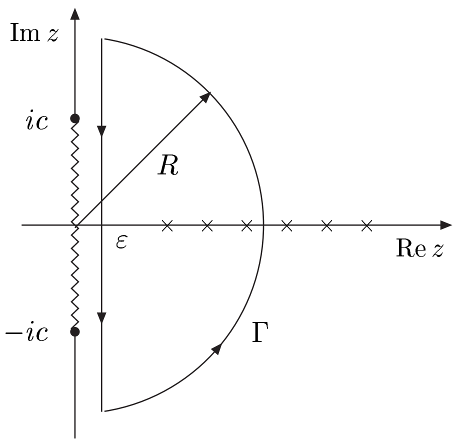

Figure 3: Contour Γ in the complex plane. The crosses along the real axis correspond to the real positive zeros x_n of the function f.

What follows applies for a generic parity, and therefore we will specify the function f only when necessary. The asymptotic expansion of $f(z)$ when $\mathrm{Im}\, z \to \pm\infty$, the same for all the parities,

$$\frac{f'(z)}{f(z)} = -\left(\pm i(z_1 - z_0) + \frac{1}{z}\right) + O\left(\frac{1}{z^2}\right) \tag{6.6}$$

tells us that the integral, like the original series, converges at infinity ($R \to \infty$) if $d < 3$. In order to find the expression of $S(d)$ for $d \to 4$, we first take $d < 3$ and extract the limit $R \to \infty$. The contribution of the integration around the semi-circle of radius R goes to zero and we are left with the vertical contour. Let us call for convenience Γ^+, Γ^- the part of this vertical contour respectively above, below the real axis. We now subtract the asymptotic behaviour of f'/f and evaluate it separately deforming Γ^+ and Γ^- to coincide with the real axis. Defining

$$F(z) = \frac{f'(z)}{f(z)} + \frac{1}{z} + i(z_1 - z_0) \tag{6.7}$$

and using the parity properties $f_{\pm\pm}(-z) = f_{\pm\pm}(z)$, $f_{\pm\mp}(-z) = -f_{\pm\mp}(z)$, we obtain:

$$\begin{aligned} S(d) = &\frac{1}{2\pi i}\left[\int_{\Gamma^+} dz\, \left(z^2 + c^2\right)^{d/2-2} F(z) - \int_{\Gamma^-} dz\, \left(z^2 + c^2\right)^{d/2-2} F(-z)\right] + \\ &+ \frac{(z_1 - z_0)}{2\sqrt{\pi}} \left(c^2\right)^{(d-3)/2} \frac{\Gamma\left((3-d)/2\right)}{\Gamma(2 - d/2)} \,. \end{aligned} \tag{6.8}$$

In the remaining integrals, we can now extract the limit $d \to 4$, being $F(z) \sim 1/z^2$ at infinity. We expand the integrand up to orders $O[(d-4)^2]$. The first term in the expansion gives a non-vanishing result because of a residue contribution in the origin (here we deform the contours Γ^+, Γ^- to coincide with the imaginary axis, $\varepsilon \to 0$ in figure 3):

$$\frac{1}{2\pi i}\left[\int_{\Gamma^+} dz\, F(z) - \int_{\Gamma^-} dz\, F(-z)\right] = \frac{\alpha}{2}\,; \qquad \alpha = \begin{cases} \pm 1 & \text{for } (\pm, \pm) \\ 0 & \text{for } (\pm, \mp) \end{cases} . \tag{6.9}$$

The second term in the expansion must be evaluated taking into account the cut along the imaginary axis between $\pm ic$. We find:

$$\frac{1}{2\pi i}\left[\int_{\Gamma+} dz\,\log\left(z^2+c^2\right)F(z) - \int_{\Gamma-} dz\,\log\left(z^2+c^2\right)F(-z)\right] =$$
$$= \log f(ic) + \log c\pi\sqrt{z_0 z_1} + \alpha\log c - c(z_1 - z_0)\,. \qquad (6.10)$$

Summing all the contributions we get our final result

$$\sum_{\{x_n\}}\left(x_n^2 + c^2(x)\right)^{d/2-2} = \frac{\alpha}{2} + \left(\frac{d}{2}-2\right)\left[\log f(ic) + \log c\pi\sqrt{z_0 z_1} + \alpha\log c\right] + O[(d-4)^2] \qquad (6.11)$$

which leads to eq. (3.3). Finally, we write the $z \to 0$ expansion of the various functions f, used in the text to obtain the low energy limit of $\Pi(p^2,\mu)$. Taking only the relevant terms, one has ($z \to 0$, $z_1 \gg z_0$):

$$\begin{aligned}
f_{++}(z) &\simeq \frac{1}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu-1}\left[\frac{4-\nu^2}{z^2 z_0^2} + \frac{2+\nu}{2(\nu-1)}\right]\\
f_{--}(z) &\simeq \frac{1}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu}\\
f_{+-}(z) &\simeq \frac{i}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu}\left[\frac{\nu-2}{zz_0} - \frac{zz_0}{2(\nu-1)} + \frac{\nu+2}{zz_0}\left(\frac{z_0}{z_1}\right)^{2\nu}\right]\\
f_{-+}(z) &\simeq \frac{i}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu}\frac{2+\nu}{zz_1}\,.
\end{aligned} \qquad (6.12)$$

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